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LETTER TO THE EDITOR

Anomalous diffusion near the flux lattice melting transition in high-T_c superconductors

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Abstract. We consider the dissipation arising from the activated motion of a flux line lattice in the presence of dilute pinning centres. By regarding this process as an effective motion of the pins relative to the flux lattice, we map the problem into the diffusion process of a particle interacting with the flux line lattice. We solve this problem analytically using a novel microscopic diffusion theory. In particular, we show that anomalous behaviour of the linear resistivity is expected at the flux lattice melting, and may show up as a dip in its temperature dependence.

One of the most important features exhibited by the novel high-T_c superconducting materials is the appearance of large dissipation due to thermally activated flux line motion [1, 2]. At low temperatures, the resistivity behaves according to the Arrhenius form $\rho = \rho_0 e^{-U_0/kT}$. Additionally, for magnetic fields parallel to the crystal axis $\boldsymbol{B} \parallel \boldsymbol{c}$, and for small values of the current j, the activation energy $U_0 = U_0(B,T)$ is independent of j [3] leading to a *linear* resistivity ρ_L, while for larger values of $j < j_c$ collective effects of flux pinning lead to a power law behaviour $U_0(j) \approx (j_c/j)^{1/2}$ [4], where j_c is the critical current. Even a more complicated dependence of U_0 on the current is possible in some cases [3, 5, 6].

The details of this resistive behaviour for $j \leq j_c$ as a function of B and T have been studied both experimentally [3, 7–9] and theoretically [10–12]. In the high-T_c materials, the low-temperature Arrhenius behaviour of the resistivity is believed to be due to thermally assisted flux flow (TAFF), where the effective motion of the flux lines is thermally activated over barriers U_0 [13]. In the simplest scenario there is a broad resistive transition from the TAFF regime to flux flow as a function of j and temperature [1, 2]. This resistive crossover transition as a function of temperature has been qualitatively explained by diffusion models based on the motion of (almost) independent flux lines or bundles in a periodic potential, with a phenomenologically fitted energy barrier $U_0 = U_{\text{eff}}(B,T)$ [11, 12]. However, this type of simplified picture of flux motion is complicated by several factors, such as collective pinning effects [5, 6, 14] and strong, microscopic pinning [2]. Another important possibility is the existence of various different phases of the flux lattice in the high-T_c materials between $H_{c_1}(T)$ and $H_{c_2}(T)$ [15–17]. In particular, the ground state Abrikosov flux line lattice (FLL) has been predicted to melt into a flux liquid [18, 19] at $H_{c_L}(T)$, where $H_{c_1}(T) < H_{c_L}(T) < H_{c_2}(T)$,

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precluding transition to the normal state at $H_{c_2}(T)$. However, an experimental verification of this transition [20] is presumably complicated by the presence of random, microscopic pinning centres in these materials. In addition, randomness in pinning has been shown [21] to destroy the true translational long-range order in the FLL, leading to a possible vortex glass phase at low temperatures for $d \geq 3$ [16]. In the case of very strong microscopic pinning, even the vortex glass-to-liquid phase transition could be destroyed, leading to a simple thermal depinning of the flux lines [2]. However, if disorder in the pinning centres is small enough and the flux lines are very rigid, a FLL can still exist up to a disorder-dependent correlation length. As a consequence, there has been no general agreement on the interpretation of some experiments which can be interpreted either as evidence of FLL melting or simply as a thermal depinning of the flux lattice. In the presence of such distinct scenarios, it would be of interest to determine possible effects on transport properties of the flux lattice melting under the assumption that the pinning centres do not completely destroy the Abrikosov melting transition.

In this letter, we shall demonstrate that the nature of the low-temperature flux phase and the Abrikosov melting have important consequences to the behaviour of the *linear* resistivity contribution ρ_L arising from the pinning of the flux lines by macroscopic defects. Namely, by considering a dilute limit of pinning centres we map the calculation of ρ_L to the diffusion of independent pins in an inhomogenous medium [14]. By using a recently developed microscopic theory of diffusion [22] we shall demonstrate that in the case of flux lattice melting, ρ_L exhibits anomalous behaviour at the transition in the form of a downward dip in its temperature dependence.

In the TAFF regime, the motion of the flux lines is assumed to be thermally activated over pinning barriers $U_0(B,T)$. Thus, the contribution to linear resistivity ρ_L arising from this process is proportional to the diffusion of the flux lines [2]. One of the theoretical approaches to this problem uses a Langevin equation to describe the flux motion in the presence of random driving forces [23]. This model of flux line diffusion has been recently applied to the case of weak collective pinning, in which (almost) independent flux lines are moving in an effective periodic potential associated with the pins [12]. The corresponding one-dimensional diffusion equation for the flux lines has been solved using multiscale analysis, yielding a simple expression for ρ_L. However, if we consider the case of quenched, dilute *macroscopic* pinning, this diffusive motion can also be described as arising from the motion of the pins relative to the FLL [14]. Thus, ρ_L becomes proportional to the tracer diffusion coefficient D_p associated with single pins interacting with the FLL. We believe that this approximation is more realistic, since with this mapping we can include the collective effects of the underlying FLL, as we shall describe below. To take this into acount, we shall consider a recently developed microscopic theory of diffusion of a particle coupled to the vibrations of an inhomogeneous medium [22]. For simplicity, we consider the case in which the magnetic field is parallel to the c axis of these highly anisotropic materials, with the vortex cores preferentially aligned along the field, and consider only the motion in the plane perpendicular to this axis. For a two-dimensional square lattice, the theory of diffusion in the high-friction limit leads to an analytic expression for diffusion coefficient as [22]

$$D_p = \frac{a^2 kT}{m} Z^{-1} \int \mathrm{d}x \left(\int \mathrm{d}y \, \mathrm{e}^{V_A(\boldsymbol{r})/kT} \eta(\boldsymbol{r}; \omega = 0) \right)^{-1} \tag{1}$$

where $a \approx \sqrt{\Phi_0/B}$ is the linear size of the unit cell, m denotes the effective 'mass' of the diffusing particle and $Z = \int e^{-\beta V_A(\boldsymbol{r})}\,d\boldsymbol{r}$ is the partition function. The two important quantities in (2) are the periodic adiabatic substrate potential $V_A(\boldsymbol{r})$ seen by the diffusing particle, and the zero-frequency limit of the friction tensor $\eta(\boldsymbol{r};\omega = 0)$ which contains the microscopic details of the coupling to the medium. Within continuum elasticity theory, $\eta(\boldsymbol{r};\omega = 0)$ is independent of temperature, and assuming a periodic Abrikosov lattice with $V_A(\boldsymbol{r}) = U_{\text{eff}}[\cos \boldsymbol{G}_1 \cdot \boldsymbol{r} + \cos \boldsymbol{G}_2 \cdot \boldsymbol{r}]$ (with $\boldsymbol{G}_1 = (2\pi/a, 0)$, $\boldsymbol{G}_2 = (0, 2\pi/a)$) and a spatially constant $\eta_c(\omega = 0)$, (1) gives the result

$$\rho_L = \rho_0 [I_0(U_{\text{eff}}/kT)]^{-2} \tag{2}$$

where ρ_0 is a constant and I_0 is a modified Bessel function of order zero. This result reproduces formally the solution of the one-dimensional diffusion problem of the flux lines [11, 12, 23]. Considering the diffusion barrier U_{eff} in (2) as phenomenological, we can immediately reproduce the results for resistivity obtained by Tinkham [11] and Inui *et al* [12] just by fitting U_{eff}. Although (2) then displays the expected Arrhenius behaviour at low temperatures, it can be considered to be an *effective fitting form* at best, since a true microscopic understanding of the associated diffusion process, and the behaviour of the energy barrier U_{eff} is lacking. Thus, instead of assuming the validity of (2) over the whole region of the resistive transition, we are going to concentrate on the case where the diffusion theory is most likely to be correct, namely *before* the crossover from the diffusive TAFF to the flux flow regime. This allows us to use the important advantage given us by our mapping to examine the *intrinsic* effects associated with the vortex lattice in detail, as we shall show below.

The microscopic expression (1) has important consequences to the motion of the flux lines near the melting transition of the FLL. To calculate the behaviour of ρ_L near the melting transition, we must consider the coupling of the pins to a realistic, triangular FLL in more detail. Although an analytic solution of the type (2) has not been found for triangular symmetry, it can be shown that D_p is still inversely proportional to the magnitude of the friction tensor $\eta(\boldsymbol{r};\omega = 0)$. In the harmonic approximation, the Fourier components of a spatial $\alpha\beta$ component of the friction tensor can be written as† [22]

$$\eta^{\alpha\beta}(\boldsymbol{G}, \boldsymbol{G}';\omega = 0) = \frac{N}{kT} \int \frac{d\boldsymbol{q}}{(2\pi)^2} \sum_{\mu,\delta} S_{\mu\delta}(\boldsymbol{q};\omega = 0) W^*(\boldsymbol{q} + \boldsymbol{G}) W(\boldsymbol{q} + \boldsymbol{G}') \times (\boldsymbol{q} + \boldsymbol{G})_\alpha (\boldsymbol{q} + \boldsymbol{G})_\mu (\boldsymbol{q} + \boldsymbol{G}')_\beta (\boldsymbol{q} + \boldsymbol{G}')_\delta \tag{3}$$

where $S(\boldsymbol{q};\omega = 0)$ is the zero-frequency limit of the dynamic structure function, $W(\boldsymbol{q})$ denotes the Fourier coefficients of the pair interaction potential with $V_A(\boldsymbol{r}) = \sum_{\boldsymbol{R}_l} W(\boldsymbol{r} - \boldsymbol{R}_l)$, $\boldsymbol{G}$ and $\boldsymbol{G}'$ denote reciprocal lattice vectors, and summation goes over spatial indices $\mu\delta$. Near a structural phase transition, such as a FLL melting, $S(\boldsymbol{q};\omega = 0)$ will display anomalous behaviour which depends on the nature of the transition. Within a third long-time approximation to structural phase transitions [24], $S(\boldsymbol{q};\omega = 0) \propto \chi^2(\boldsymbol{q})$, where $\chi(\boldsymbol{q})$ is the static susceptibility. Thus, in the case of a continuous transition $\chi(\boldsymbol{q}_0) \approx t^{-\gamma}$, i.e. the susceptibility diverges near $t = T - T_m = 0$

† Although we are considering single particle diffusion here, this expression for the friction tensor takes into account collective effects of the vortex lattice via $S(\boldsymbol{q};\omega = 0)$. A somewhat similar expression for the random component of the velocity of flux lines in the case of weak, collective pinning has been recently derived by Vinokur *et al* [14].

at a value of $\boldsymbol{q}_0$ corresponding to the commensurate wavevector of the triangular FLL. Neglecting the contribution from the other smooth functions of $\boldsymbol{q}$ in (3), and using an isotropic mean-field scaling form for $\chi(\boldsymbol{q}) \propto 1/(\boldsymbol{q}-\boldsymbol{q}_0)^2 + t^\gamma$ gives $\eta(\boldsymbol{G}, \boldsymbol{G}'; \omega = 0) \propto t^{-\gamma}$ which implies

$$\rho_{\mathrm{L}} \approx D_{\mathrm{p}} \approx t^\gamma \tag{4}$$

for $t \to 0$ at $d = 2$. Thus, we have the following novel prediction: due to a diverging friction tensor, the linear resistivity component associated with macroscopic pinning vanishes at T_{m}. A more careful calculation using a full scaling form of the dynamic structure function gives $D_{\mathrm{p}} \approx t^{(z+2-d-\eta)/(2-\eta)\gamma}$ [25], where η and z are the correlation function and dynamic critical exponents, respectively†. We note that the mean-field limit at $d = 2$ where $z = 2$, $\eta = 0$ captures back the result (4). Physically, a reduction in the diffusion constant is expected since there should be an increase in the effective pinning due to a softening of the flux lattice by thermal fluctuations at the transition [2].

Recently, a qualitatively similar prediction about the behaviour of the linear resistivity component ρ_{L} at a *vortex glass transition* of a FLL has been presented by Koch *et al* [26] using simple scaling arguments. In their theory, they obtain $\rho_{\mathrm{L}} \approx t^{\nu(z+2-d)} \approx t^{(z+2-d)/(2-\eta)\gamma}$ above T_{m}. In the vortex glass phase below T_{m}, $\rho_{\mathrm{L}} \equiv 0$, in contrast to the weakly pinned FLL, for which we assume ρ_{L} to display activated behaviour. Their result has been derived by assuming that the relevant length and time scales at the transition are controlled by the coherence length ξ and ξ^z, respectively. In the case we are considering here, however, the resistivity near T_{m} is assumed to be controlled by the *correlation function of the* FLL in the friction tensor, leading to a different result. For $\eta = 0$, our result coincides with that of Koch *et al.* However, we must emphasize the different physical regime that we are working on: we are neglecting the collective pinning effects which lead to the vortex glass phase, and we are assuming that ρ_{L} is finite *both* above *and* below the melting transition.

The behaviour of ρ_{L} near the FLL melting crucially depends on the nature of the melting transition. In case the true long-range order of the FLL is destroyed by random pinning, the diverging correlation length is limited by the distance over which short-range order remains and we expect the transition will cause a dip in ρ_{L} at T_{m}, instead of making it vanish. The same situation also applies, if the FLL melting is a first-order transition. The stronger the first-order nature of the melting is, the weaker the dip exhibited in ρ_{L}. Above T_{m}, the behaviour of the resistivity is complicated by pinning. For weak, short-range pinning it has recently been shown [14] that the TAFF regime may extend up to a temperature $T_k > T_{\mathrm{m}}$, beyond which it crosses over to the flux flow regime. For stronger pinning, the flux flow regime may also start at around T_{m}. In the latter case, our prediction only holds for $T \leq T_{\mathrm{m}}$.

So far, the available experimental results of resistivity measurements in an applied magnetic field have not shown the behaviour we have discussed in this work. This is probably due to the difficulties of performing resistivity measurements in the ohmic regime. The requirement of extremely small voltages may be diffult to achieve near transition and non-linear effects probably dominate in most experiments. Alternately, in some high-T_{c} materials microscopic pinning may be strong enough to destroy the melting transition, as we have discussed above. However, in granular superconducting

† In some cases the true singularity resulting from the divergence of $S(\boldsymbol{q}; \omega = 0)$ may be considerably weaker than the predictions obtained here [27].

materials, as well as in some Josephson junction arrays, it may be possible to control both resistivity and pinning barriers in order to observe a melting transition of the flux lattice. We hope that the present work will motivate further experimental studies in these directions.

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